puter solution gives 0.390. For other values of t/R, the minimum $\sigma R/Et$ and the corresponding values of a, b, η, μ, ω, and $\epsilon R/t$ are given in Table 1. It is seen that the minimum $\sigma R/Et$ now depends on t/R, whereas in Ref. 2 this dependency does not exist. The influence of t/R on $\sigma R/Et$ is shown in Fig. 2. For $t/R = 0.03$, the minimum $\sigma R/Et$ has a 4.5% increase over that at $t/R = 0$.

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Shock Shape Generalization for Inverse Blunt Body Methods

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THE purpose of this note is to set forth a concept and technique for generalizing shock wave geometries while maintaining the feature of equal spacing between finite difference points in determinate form. This concept is useful to the solution of inviscid flow fields around blunt noses in supersonic or hypersonic flight, particularly in the case of the inverse or shock-initiated approach.

In essence, the inverse blunt-body approach starts with an assumed shock shape and applies finite difference techniques to the equations of motion (conservation of mass, momentum, and energy), marching progressively inward from the shock wave to the body. The body contour itself is established by this process, and is hoped to be close to a specified contour of practical interest. In many cases, particularly when the desired body contour is circular, the bodies attainable from simple shock shapes are close enough to the desired contour for some engineering purposes. However, there occur at least two situations in which more general shock shapes are needed: a) when greater accuracy in body contour matching is desired for more refined predictions of local properties; and b) when simple shock shapes fail to provide data sufficiently far around the body to adequately envelop the sonic line and supply direct input lines for method of characteristics programs.

Generalization from simple shock shapes to a family of conic section shock shapes was achieved by Van Dyke,[1] using a special orthogonal coordinate system. The program of Refs. 2 and 3 contains a one-parameter capability for modifying shock shapes from basic conic section shock geometries. These are significant steps in the direction of shock shape generalization.

The finite-difference processes customarily applied involve the numerical differentiation of flow properties along the shock wave (or, subsequently, along lines displaced therefrom across known finite steps). The numerical difference formulas that are most efficiently applicable require that the data points at which properties are known be equally spaced with respect to some spacing parameter, usually one of the coordinates in some geometric coordinate system. The reference circle coordinate system of Ref. 4 (Fig. 1 herein), somewhat independent of shock shape, uses the reference circle arc length (or radial angle) as a spacing parameter applied through the difference formulas of Ref. 5.

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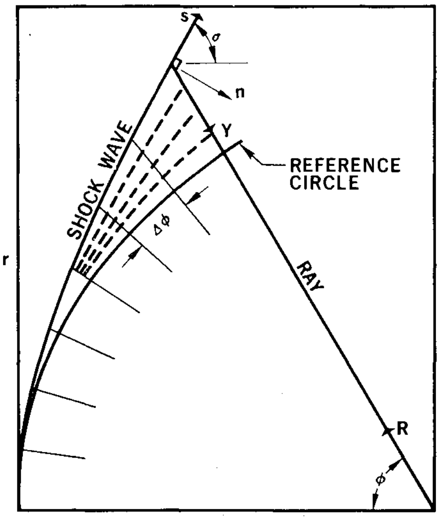

Fig. 1 Reference circle system.

Reference Circle Method Geometric Development

The reference circle concept of Ref. 4 was developed to provide an axis system somewhat independent of shock shape, capable of scale changes, and applicable to cases in which shock arc lengths are not easily determined. The simplest and most limited application of the reference circle method is to let the shock wave coincide with the reference circle, in which case the curvilinear equations of motion with basic variables of P, ρ, u, and v (Refs. 4 and 6) can be applied without any geometric adjustments.

Next in order of difficulty is the operation of the reference circle system initiated from a simple analytic, noncircular shock shape. The geometric consequences of this advance are portrayed in Fig. 1. The objective is to march from the shock wave to the body along rays of constant ϕ, spaced apart by constant intervals $\Delta\phi$. Once the reference circle is reached, the process is identical to that behind a circular shock wave. Operations in the front-end region will now be discussed.

First of all, the intersections of the rays with the shock wave are determined. An exact analytic solution for these intersections with conic section shock shapes is presented in Ref. 4; a convergent iterative solution for catenary geometry is also given.

The determination of the (x, r) coordinates at all shock-ray intersections leads immediately to the local geometric slopes at each point dr/dx. These could often be found as exact analytic expressions; however, for the purpose of generality, they are found through the difference formulas as $(dr/d\phi)/(dx/d\phi)$. The local shock angles, $\sigma = \tan^{-1}(dr/dx)$, together with given conditions in front of the shock wave, determine conditions immediately behind the shock wave at each local point. Another useful geometric parameter found with the aid of the difference formulas is

$$ds/d\phi = [R^2 + (dR/d\phi)^2]^{1/2} \qquad (1)$$

The shockwise derivatives are then simply

$$df/ds = (df/d\phi)/(ds/d\phi) \qquad (2)$$

where f is any property or velocity component (P, ρ, V_n, V_s).

The shockwise derivatives df/ds constitute partial derivatives in a local (s, n) curvilinear axis system composed of the shock wave and a straight line normal thereto through the local point of interest. Application of the equations of motion in this local system produces partial derivatives in the n direction. Vectorial considerations determine the transformation of the s derivatives and n derivatives into Y derivatives, which are the radially directed gradients sought. Details concerning the vectorial transformations are presented in Ref. 4.

In the preceding discussion, the line of interest has been the shock wave. Once the shock-wave point locations have been established, other lines composed of points located on rays at specified fractions of the distance between shock wave and reference circle are likewise established. The treatment of these lines is identical to that of the shock wave, except that the properties along the line come from the forward marching process initiated at the shock wave.

It is noteworthy that, although these intermediate grid lines may not generally be expressed as simple analytic functions, such expressions are not required for the application of the equations of motion. All that is needed is the local properties and geometric coordinates at points spaced equally with respect to a geometric spacing parameter. The difference formulas and the principles of coordinate transformation provide the necessary geometric operations. The procedures followed do not depend upon the form of the grid line.

Advancing this concept one step further, it is clear that the shock wave itself need not be a simple analytic expression. To maintain the necessary smoothness the shock wave should be analytically determined, but many analytic combinations can be devised to provide a great variety of shock shapes. In Ref. 4, examples are given for shock shapes defined simply by conic sections and then adjusted by adding radially (in the R direction) a term $a\phi^k$, where k is an even integer for axial symmetry. Such adjustments were found useful in more closely matching body contours and in extending body coverage out to local Mach numbers high enough for method of characteristics operations without resorting to extrapolation.

The idea of shock-shape adjustment has been carried even further. In an effort to very closely reproduce a body contour of interest, a series of adjustment terms has been studied. The form is

$$\Delta R = a\phi^2 + b\phi^4 + c\phi^6 + \ldots \quad (3)$$

The coefficients a, b, c, etc. were determined by an inverse matrix procedure utilizing deviations of program-produced body contours from the desired body contour as determined from cases applying selected values of a, b, c, etc., separately. Although some success was attained with this generalization, the higher accuracy required for the matrix operations often was found to approach or exceed the 8-digit accuracy supplied by the IBM 7094 in single precision operations. Further explanations and results of this extension appear in Ref. 4.

Discussion

Examples of body contour accuracies attainable from shock shape generalization are presented in Ref. 4. A useful accuracy criterion is the root-mean-square deviation $\bar{d}$ of the program-produced body contour from its sought-for exact counterpart, scaled and fitted to the program-produced body points by the method of least squares. Using a reference circle radius of 1.0, nearly spherical bodies of radii in the neighborhood of 0.75 are produced at flight Mach numbers of 6 and above. For such cases, deviations $\bar{d}$ computed along reference circle rays for optimum (i.e., minimizing $\bar{d}$) conic section shock shapes at Mach numbers of 6, 10, and 10^4 were 0.000157, 0.000130, and 0.000161, respectively. Corresponding $\bar{d}$ values obtained from shock shapes defined by a circle modified by ΔR values of $a\phi^4$, with a optimized, were improved to 0.000068, 0.000034, and 0.000017, respectively. All of the cases quoted produced reliable body coverage (data points) out to local Mach numbers somewhat beyond 1.1. Even greater improvements are attainable through shock-shape generalizations beyond the $a\phi^4$ type. Indeed, in the case of the reference circle method, shock-shape generalization beyond conic sections alone has been found an indispensable tool in defining flow fields around spherical bodies at low flight Mach numbers and around circular two-dimensional bodies. Recently, some highly accurate results were obtained for very blunt ellipsoid bodies (with body bluntness parameters B up to 4).

In summary, many varied shock-wave shapes have been generated to initiate one version of the inverse approach to the flow around a blunt body. It appears that similar geometric generalization techniques can also be applied to broaden the scope of other versions of the inverse approach.

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Experimental Determination of Velocity Lag in Gas-Particle Nozzle Flows

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Nomenclature

A	= cross-sectional area of flow for particles
d	= pathlength
n_1,n_2	= real and imaginary parts of the complex index of refraction
$N(r)$	= number of solid particles in size range Δr centered about radius r per unit volume of gas-particle mixture
$\tilde{N}$	= number of solid particles of all sizes per unit volume of gas-particle mixture
p	= pressure
$Q^{(e)},Q^{(s)}$	= efficiency factors for extinction and scattering (ratio of extinction or scattering cross section to the geometrical cross section)
r	= particle radius
R	= spectral radiancy
T	= temperature
u	= particle speed
v	= gas speed
$\bar{w}$	= average weight for particle size distribution

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